The Algorithmic Lens:
How the Computational Perspective is Transforming the Sciences

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What is Computer Science?

"CS is the only scientific discipline that cannot be defined in a single sentence"

(now seriously folks…)
What is Computer Science?

- Applied science?
- Engineering discipline?
- Branch of Math/Physics/EE?
- *"The study of information and its algorithms"?*
- (in the era of the Internet): Also a natural and social science?

What is Computer Science? (cont.)

- Mathematics: *"The queen and servant of sciences"*
- Queen: Power, authority, pride
- Servant: Influences and transforms by being useful, powerful, and universal
- My point: CS is the new math

The evidence: Eight vignettes from

- Mathematics
- Physics
- Biology
- Economics and Social Science

(with many thanks to…

- Dick Karp, Alistair Sinclair, Umesh Vazirani, Elchanan Mossel, Scott Shenker
- The SIGACT Committee for the advancement of theoretical CS)

The queen's crown jewels:

The seven Clay Institute Millennium problems

- Birch and Swinnerton-Dyer Conjecture
- Hodge Conjecture
- Navier-Stokes Equations
- ***P vs NP***
- Poincaré Conjecture
- Riemann Hypothesis
- Yang-Mills Theory

"the deepest, and most fundamental and consequential, open problem in Mathematics today is not about geometry or whole numbers: it is about ***computation***…"

The queen is snobbish

- Mainstream Mathematics has been much more reserved about other fields, such as Foundations/Logic, Mathematical Physics, and Combinatorics
- P *vs* NP was adopted by mathematicians only after three decades of powerful work by computer scientists failed to solve it

P *vs* NP:
the quest for depth

btw: NP-Completeness

- CS's most successful intellectual export

Perturbing Physics

The computational worldview provides new insights into, ***and tests***, some of the most prestigious theories about the universe

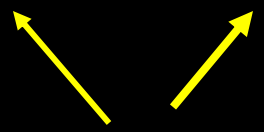
Quantum computation: reinventing the bit

bit: a wire can have
- many electrons
- few electrons

qubit: an electron can be
- close to the nucleus
- far

Big difference

- A bit is either 0 or 1
- A qubit is in *both states* $Q = \alpha |0\rangle + \beta |1\rangle$



complex numbers
"probabilities"

- An n-qubit system is in 2^n states at the same time!

Three possible reactions

1. How curious, Nature is extravagant!

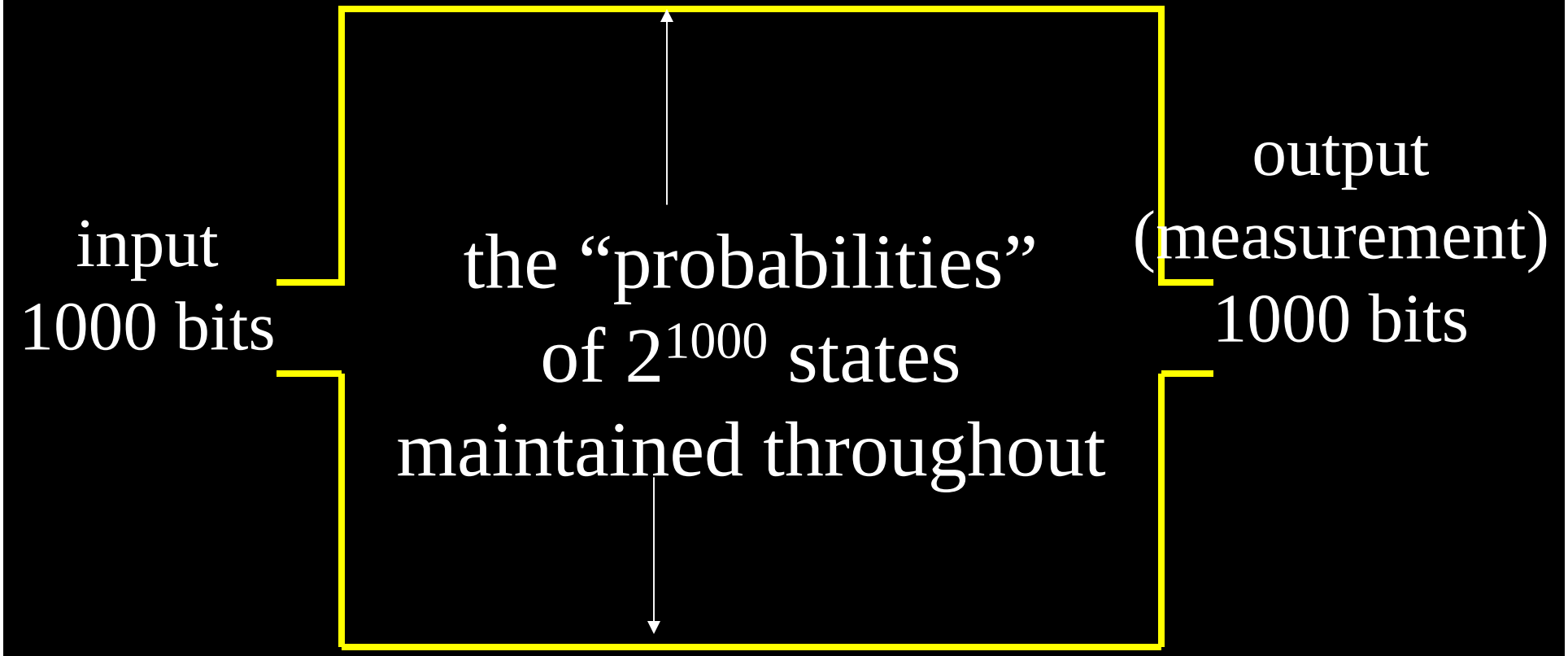
   (remember Einstein: "The Old One does not throw dice")

2. Oh my God, how do you simulate such a system on a computer?

3. ***But what if we built a computer out of these things?***

How to factor a 1000-bit integer

in ~1000 easy steps



But can we build these computers?

The three eventualities

1. Yes!
2. No, because of a thousand annoying little problems and details (plus, eventually, lack of funding…)
3. ***No, because Quantum Physics holds only for tiny numbers of particles!!!***

"Quantum computation is as much about testing Quantum Physics as it is about building powerful computers."

Umesh Vazirani

Also: Statistical Mechanics

- In physical systems, when parameters of local interactions evolve, macroscopic properties change dramatically, and we have a *phase transition*
- Certain randomized algorithms are known to converge *exponentially faster* when the parameters are in the right range
- Deep fact: *These two phenomena are identical*

Statistical Mechanics (cont.)

- The traffic is two-way: ***Belief propagation*** and ***survey propagation*** algorithms for constraint satisfaction
- Phase transitions in the Internet and the www?

Disrupting Biology

- Several mysteries in Biology can be productively approached as algorithmic problems, e.g.
- evolution
- the brain
- the immune system
- …and many more

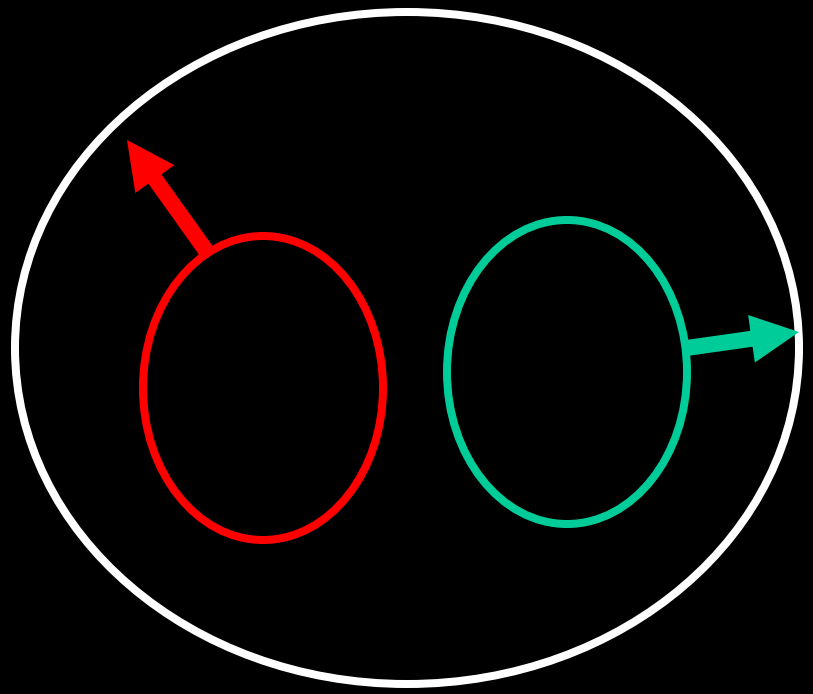
The mystery of internal conflicts

We often find ourselves in situations of *internal conflict*

- Between our conscience and our ambition
- Between our duty and our laziness
- Between our appetite and our diet
- ***Is this a way to run a brain?***

What is an internal conflict?

- Two subsystems
- Their behavior is best described in terms of two objective functions
- ***These functions do not align***



Can an optimal system have an internal conflict?

Theorem [Livnat-Pippenger 2006]:

Yes, if complexity is taken into account

The mystery of evolution

"To think that the eye could evolve by natural selection seems, I freely confess, absurd to the highest degree"

Charles Darwin 1859

The computational perspective does not seem to help:

"How do you search for a 3×10^9-long string in 3×10^9 years?" Les Valiant 2007

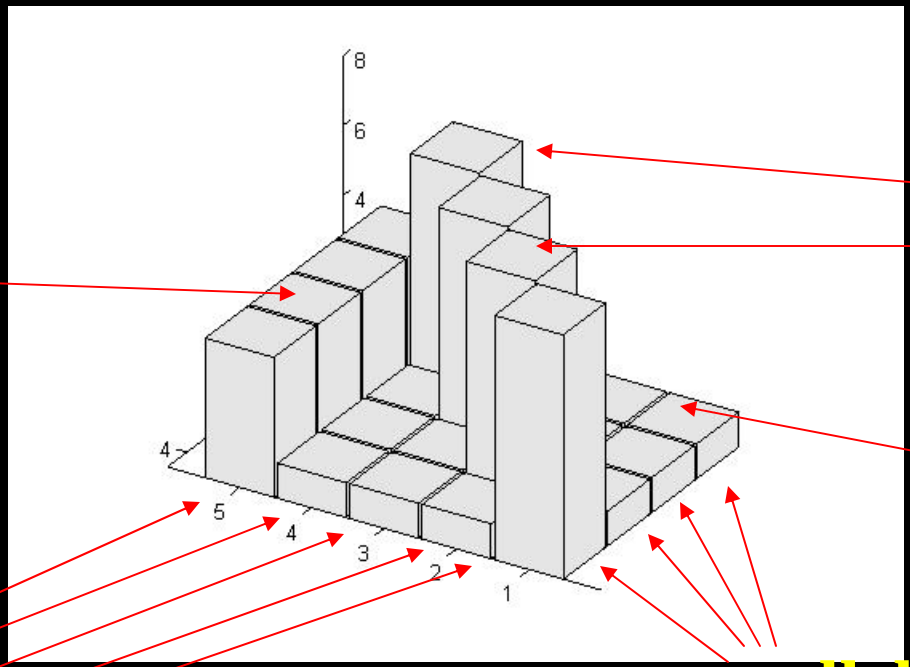
A seemingly unrelated question:

- Why does simulated annealing work so much better than genetic algorithms in solving optimization problems?

simulated annealing $\Leftrightarrow$ asexual reproduction

genetic algorithms $\Leftrightarrow$ sexual reproduction

Fitness landscapes: “peaks” *vs* “plateaus”



“plateau”

peaks

troughs

alleles (variants) of gene A

alleles of gene B

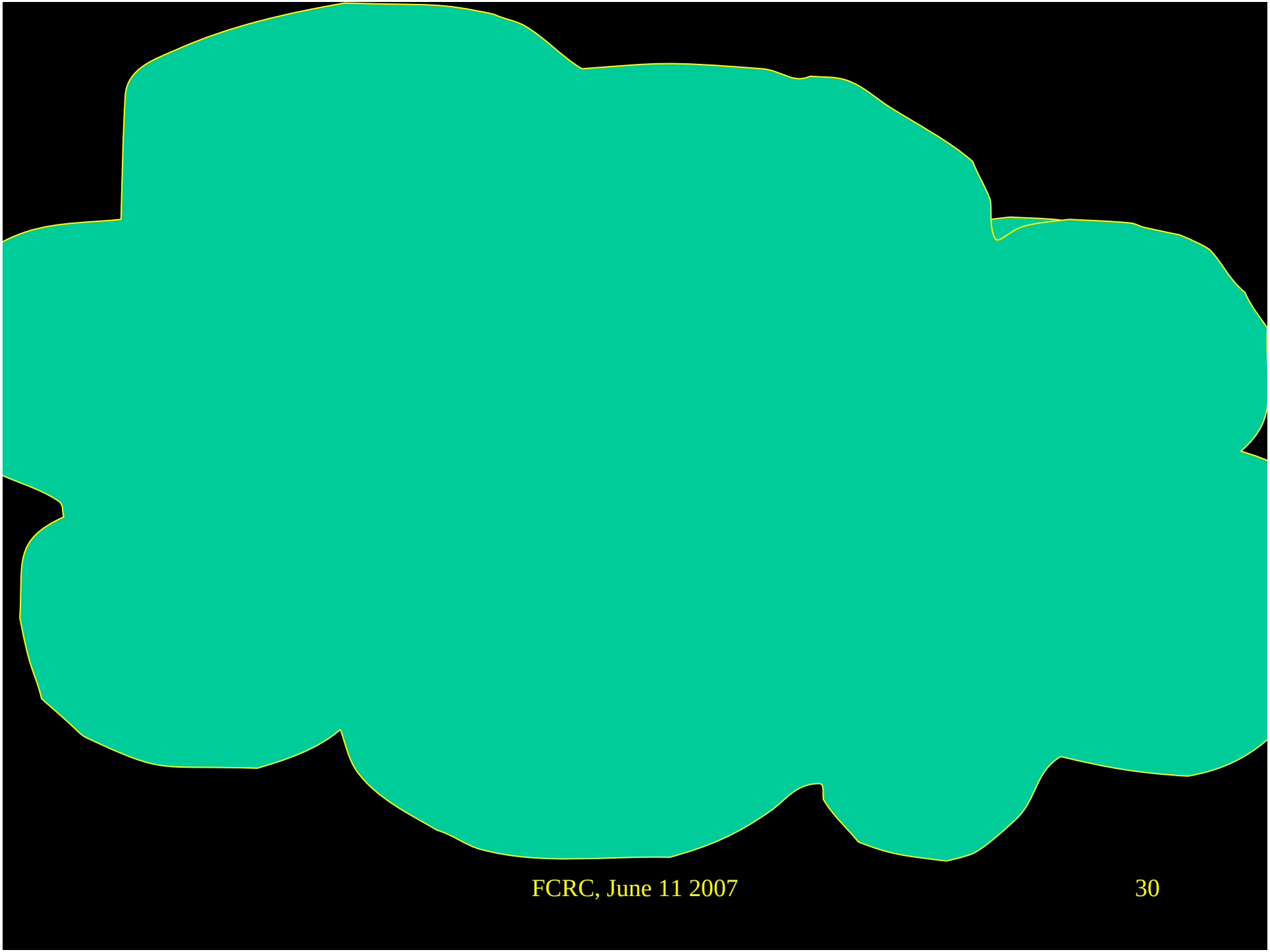
Sex favors plateaus over peaks!

Theorem [Livnat & P, 2007]:

- Unless peak > 2 × plateau, in sexual reproduction the plateau will dominate and the peaks will become extinct
- In asexual reproduction, the peaks will always dominate and the plateau will become extinct

And plateaus accelerate evolution

- They act as springboards allowing alternatives to be explored *in parallel*…
- …and this acceleration promotes *speciation* (the creation of new species)…
- …which results in an altered landscape…
- …in which sex selects more plateaus…
- …and life goes on…



The Internet turned CS into a natural science

The first computational artifact that was ***never designed***, and hence must be approached by the *scientific method:*

- Observations
- Experiments
- Falsifiable theories
- Specialized applied mathematics

…and a social science

- Economics and Game Theory

  "The Internet is an equilibrium, we just have to identify the game"

  Scott Shenker

- Sociology

  The Internet cannot be studied in isolation from the complex social system it enables and serves. And it is an ideal test bed for sociological analysis and experimentation.

Behavior predictions in Economics: Equilibria

- Nobody has an incentive to change, as long as everybody else stays put

The story of equilibria

[von Neumann 1929]: They always exist, as long as the game is two-player zero-sum

[Nash 1951]: Even in nonzero-sum, multiplayer games

[Arrow-Debreu 1952]: In markets too

Question: Can they be computed efficiently?

But why should we care about algorithms for equilibria?

- Equilibria are predictions of behavior
- *Computational tractability is an important modeling prerequisite*

  "If your laptop can't find it, then neither can the market." Kamal Jain
- Important CS contribution to the debate on solution concepts in Economics

…and very recently…

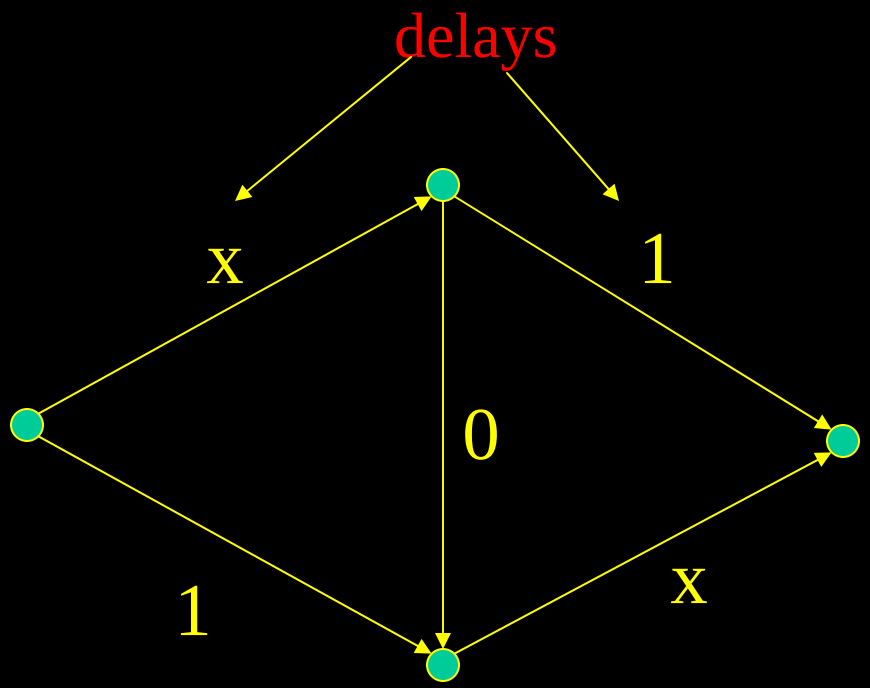
Theorem [Daskalakis, Goldberg & P, 2006]:

Finding a Nash equilibrium is an intractable problem

Also, equilibria are inefficient: "The price of anarchy"

$$\text{p. of } (A) = \frac{\text{cost of worst equilibrium}}{\text{"socially optimum" cost}}$$

Selfishness can hurt you!



Social optimum: 1.5

Selfish equilibrium: 2

…but [often] not too much

Theorem [Roughgarden and Tardos, 2000]:

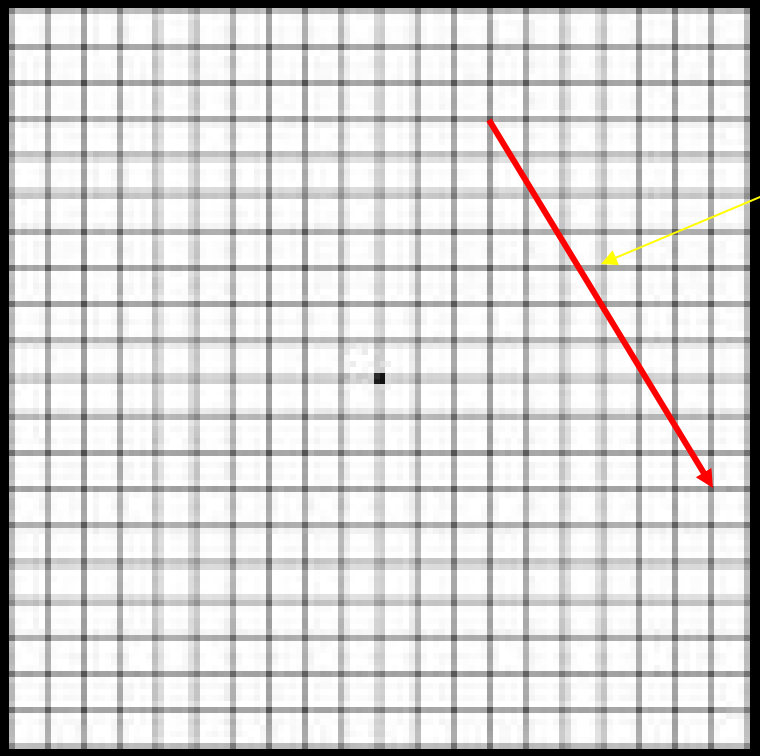
Price of anarchy = 4/3

Finally, sociology under the lens:
the Milgrom 1968 experiment, revisited

It's a small world:
"six degrees"
on average

Q: for which networks does this work?

Who do you know? (besides your neighbors…)



Prob = $[\text{distance}]^{-A}$

Theorem [Kleinberg 2001]

- If $A = 2$, $\log^2 n$ hops
- If $A \neq 2$, n^B hops

Another convergence

- Social networks become large, explicit and available
- Their analysis is informed by the same algorithmic applied math that was developed for understanding the Internet

So…

- The algorithmic worldview is changing the sciences: mathematical, natural, life, social
- CS is placing itself at the center of the scientific discourse and exchange of ideas
- And this is only the beginning…

Thank you!